Nucleation fields in composite medium grains

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(Received 12 September 2007; accepted 25 October 2007; published online 24 January 2008)

An expression for the nucleation field from initial saturation of magnetization reversal is derived for a two-layer magnetic system with variable interlayer exchange coupling. Nonuniform domain wall-like rotations are considered in both films. A relatively simple relation occurs when magnetostatic interactions are neglected and the reversal field is applied along the particle symmetry axis. The results are compared with numerical micromagnetic solutions that include magnetostatic interactions. Good agreement is obtained for a range of parameters, especially when the interlayer exchange is chosen to minimize the system coercivity. Expansion of the model to more than two interacting layers is discussed. © 2008 American Institute of Physics. [DOI: 10.1063/1.2833312]

I. INTRODUCTION

Current high density magnetic recording systems are comprised of multilayer media in which the anisotropy axes of each layer are oriented perpendicular to the film plane. Numerous numerical and analytical solutions to magnetization reversal have been published.1–5 Here, following Ref. 6, an implicit expression for the field from initial saturation of a dual layer magnetic grain is determined. A nonuniform reversal is allowed in each layer. Magnetostatics is neglected, an adjustable interlayer exchange is included, and the applied field is taken to be parallel to the particle symmetry axis. Because only nucleation is being evaluated, small angle approximations to the system energy are considered. In general, the extent of nonuniform rotation for either layer depends on the ratio of the domain wall length to the layer thickness. Typically, these composite media have a “soft” and a “hard” layers. For “exchange spring” systems, where the soft layer is relatively thick, a nonuniform rotation can occur (and is designed) into the soft layer.

II. MODEL

A schematic of the dual layer system is shown in Fig. 1. Two subgrains labeled with subscript 1,2 comprise a single grain. The uniaxial anisotropy constants \( K_1, K_2 \) have common axes along the particle length direction. The sublayers have saturation magnetizations, exchange levels, and layer thicknesses of \( M_1, M_2, A_1, A_2 \), and \( t_1, t_2 \), respectively. \( J \) is a variable interlayer exchange (energy per area). Because magnetostatics is not included, the grain cross-sectional shape (noted by diameter \( D \)) does not enter. Magnetization orientation variations only along the particle axis are considered \( \theta_1(z), \theta_2(z) \). Here, for convenience, \( z = 0 \) in each layer will be at the common boundary; \( z = t_1, z = t_2 \) represent the outer boundaries of each sublayer. The applied field \( H \) (positive in the reverse direction from initial saturation) is along the grain symmetry axis.

The system energy (per unit cross-sectional area) is

\[
E = \int_0^{t_1} \, dz \left[ \int_0^{t_2} \, dz \left( A_1 \left( \frac{\partial \theta_1(z)}{\partial z} \right)^2 + K_1 \sin^2 \theta_1(z) \right) + HM_1 \cos \theta_1(z) \right] + \int_0^{t_2} \, dz \left[ A_2 \left( \frac{\partial \theta_2(z)}{\partial z} \right)^2 + K_2 \sin^2 \theta_2(z) + HM_2 \cos \theta_2(z) \right] - J_z \cos \theta_1(0) - \theta_2(0). \tag{1}
\]

In Eq. (1), we assume that the magnetization rotates in a plane with no azimuthal rotation about the \( z \) axis. In order to use scaled parameters, the energy is normalized by \( 2K_1t_2 \). In addition, the energy integrals are written using the scaled parameters \( u = z/t_1, v = z/t_2 \) in the respective regions.

![FIG. 1. Geometry of dual layer grain. The magnetization is allowed to rotate along the grain length. The coordinates \( z \) in each layer have a common origin at the interface.](Image)
\[ E_n = \frac{E}{2K_{2f}} = \int_0^1 du \left\{ \frac{k}{2} \left[ \frac{\delta_1 \partial \theta_1(u)}{\partial u} \right]^2 + \frac{k}{2} \sin^2 \theta_1(u) + hm \cos \theta_1(u) \right\} + \int_0^1 du \left\{ \frac{1}{2} \delta_2 \partial \theta_2(u) \right\}^2 + \frac{1}{2} \sin^2 \theta_2(u) + h \cos \theta_2(u) \right\} - J \cos[\theta_1(0) - \theta_2(0)]. \]

In Eq. (2), the scaled parameters are \( h = H/H_{K_2}, \) \( m = M_1 t_1/M_{2f}, \) \( k = K_1 t_1/K_{2f}, \) \( J = J_1/2K_{2f}, \) \( \delta_1 = \sqrt{A_1/K_1}, \) and \( \delta_2 = \sqrt{A_2/K_2}. \) In practice, systems of interest occur where one layer is soft with respect to the other hard layer so that \( K_1 < K_2 \) or \( k < 1. \)

Standard variational techniques are utilized (e.g., Ref. 7). A second order differential equation with (coupled) boundary conditions is obtained in each layer by setting the variation of the total energy to zero and using the chain rule on the variation of the exchange energy: \( \delta \partial \theta_1(u)/\partial u \right\}^2 / 2 = \left[ \delta \partial \theta_1(u)/\partial u \right] \partial \theta_1(u)/\partial u - \left[ \delta \partial \theta_1(u)/\partial u \right] \delta \partial \theta_1(u)/\partial u \]. One of the chain rule terms integrates immediately to give boundary conditions. The resulting differential equations in a small angle form with boundary conditions are as follows:

\[ \frac{\partial^2 \theta_1(u)}{\partial u^2} = \theta_1(u)(1 - hm/k)(\delta_1/t_1)^2, \]

(3)

\[ \frac{\partial^2 \theta_2(u)}{\partial u^2} = \theta_2(u)(1 - h)(\delta_2/t_2)^2, \]

(4)

\[ \frac{\partial \theta_1(1)}{\partial u} = 0, \]

(5)

\[ \frac{\partial \theta_2(1)}{\partial u} = 0. \]

(6)

Only the boundary conditions at \( \theta_1(1) \) and \( \theta_2(1) \) are needed to obtain the nucleation field because equilibrium prior to nucleation is known \( \theta_1(u) - \theta_2(u) = 0 \). The solutions to the differential equations in terms of \( \theta_1(0), \theta_2(0) \) are (see, e.g., Ref. 6)

\[ \theta_1(u) = \theta_1(0) \frac{\cosh[(t_1/\delta_1)(1 - u)]1 - hm/k]}{\cosh[(t_1/\delta_1)1 - hm/k]}, \]

(7)

\[ \theta_2(u) = \theta_2(0) \frac{\cosh[(t_2/\delta_2)(1 - u)]1 - h]}{\cosh[(t_2/\delta_2)1 - h]}, \]

(8)

These equations apply for all values of \( h \geq 0 \): When the square roots take on imaginary values, we can simply replace the cosh by cos and use the absolute values of the quantities under the square root signs. Note also that these equations are symmetric because \( hm/k = H/H_{K_1} \).

Examples of the angular variations [Eqs. (7) and (8)] are shown in Fig. 2. For illustration, we take \( \theta_1(0)/\theta_2(0) = 1.5 \) and plot the initial (relative) deviation from equilibrium for different values of normalized “wall” thickness in each layer. In general, the largest deviation is at the top surface of the soft layer \( k < 1 \); the deviation continuous decreases along the grain to a smallest value at the bottom of the hard layer. Note that for small \( K_1 \), as in a soft layer, \( \delta_1/t_1 \) is large and the angular variation is small. For a large \( K_2 \), as in a hard layer, \( \delta_2/t_2 \) is small and the angular variation can be relatively large. Thus, for a composite film with a thin soft layer, the predominant nonuniform rotation occurs in the hard layer (e.g., in Fig. 2 for \( \delta_1/t_1 = 2, \delta_2/t_2 = 0.25 \)). However, for an exchange spring with a thick soft layer, the rotation in the soft layer (layer 1) can be large.

Substitution of the solutions [Eqs. (7) and (8)] into the energy [Eq. (2)] gives the energy simply as a two variable problem in terms of \( \theta_1(0), \theta_2(0) \). For small field values, the energy increases for all finite values of \( \theta_1(0), \theta_2(0) \). The nucleation field arises when a direction occurs in the \( \theta_1(0), \theta_2(0) \) space where the energy decreases. Expanding the energy to second order and setting the determinant to zero yields an expression for the nucleation field,

\[ \left[ \frac{(k-hm)\tanh \alpha_1}{\alpha_1} + J \left[ \frac{(1-h)\tanh \alpha_2}{\alpha_2} - J \right] \right] = 0, \]

(9)

where \( \alpha_1 = (t_1/\delta_1) \sqrt[1-hm/k], \)

\( \alpha_2 = (t_2/\delta_2) \sqrt[1-h]. \)

Solving Eq. (9) gives an analytical expression for \( J(h_{nuc}) \). To numerically invert this solution, we tabulated \( J(h_{nuc}) \) and constructed the inverse for plotting. Note that we can introduce additional layers and interfaces. For each layer, a differential equation is obtained with boundary conditions at the
adjacent interfaces. For \( n \) layers, an \( n \times n \) determinant will arise.

### III. COMPARISON WITH MICROMAGNETIC SIMULATIONS

In Fig. 3, the numerically simulated \( M-H \) loops are shown for three values of interlayer exchange. The simulations include full magnetostatics. In terms of the scaled parameters to be used in Eq. (9), \( m=1, k=0, t_1/\delta_1=0, t_2/\delta_2=2.44 \) [or explicitly for \( t_1=t_2 \) and \( A_1=A_2, \alpha_1=(t_2/\delta_2)\sqrt{h} \)]. Using Eq. (9), the calculated nucleation fields are \( h_n=H_n/H_{K2}=0.08, 0.13, 0.165 \) for \( J=0.1, 0.2, 0.3 \), respectively, as shown by the “dots” in Fig. 3. Even though magnetostatics is neglected in Eq. (9), the agreement is quite good, especially for the larger \( J \) values that give the minimum coercivity. Good agreement is obtained because, in this example, each layer thickness is equal to half the grain diameter and the effect of magnetostatics is expected to be small.

Another example is illustrated in Fig. 4 (unscaled). In this case, the grain is long with respect to the grain diameter so that the effect of magnetostatics can be significant \( (t_1/D=t_2/D=2) \). Each layer has the same saturation magnetization \( (M_1=M_2) \) equal to 318, 637 G for each case. In layer 1, \( K_1=0 \) so that \( k=0 \). In layer 2, the field values come from \( K_2=9 \times 10^6 \) erg/cc. The intrinsic exchange is the same in each layer: \( A_1=A_2=1.1 \times 10^{-6} \) erg/cm. For \( t_1=t_2=12 \) nm, the scaled interlayer exchange ranges from \( J \approx 0.05 \) to 5. In the analytical model, the shape anisotropy term is added to layer 1 because rotation in that layer was virtually uniform. The agreement is reasonable, although not as good as for the smaller nearly cubic grain in Fig. 3.

### IV. CONCLUSION

The nucleation field for magnetic reversal is calculated for a dual layer magnetic grain. The analysis allows for non-uniform domain wall-like magnetization variations along the grain symmetry axis. The nucleation field involves only small angle deviations from saturation and, thus, a relatively simple solution occurs even for the combination of exchange, anisotropy, and Zeeman energies. Results are compared with numerical solutions that include magnetostatic fields. The agreement is quite good showing that the effect of magnetostatics in the examples shown is not large. The technique may be extended to any number of \( n \) layers, although the solution involves the vanishing of a \( 2n \) determinant. In this manner, graded anisotropy systems may also be analyzed.

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