ECE 222b
Applied Electromagnetics
Notes Set 4a

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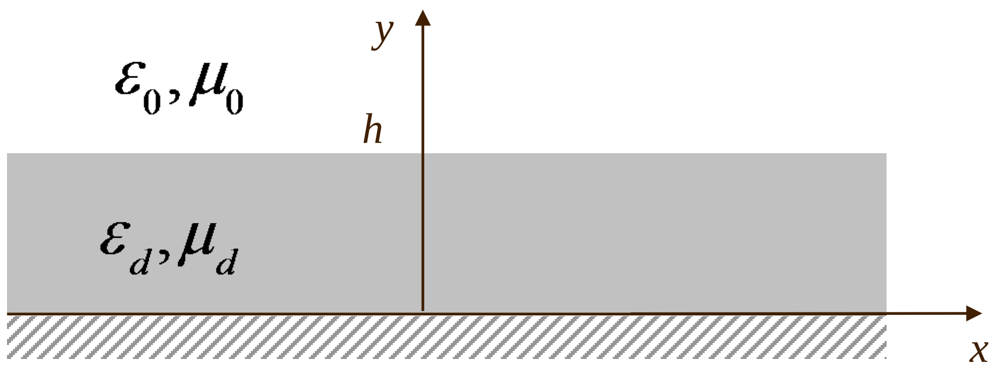
Dielectric Slab Waveguide (1)



Assumptions: 1. No variation along *x*

2. Wave propagation in the *z* direction

Examine the equation for inhomogeneous waveguides:

$$\nabla_t \times \left[\frac{1}{k_t^2} (\omega\varepsilon \hat{z} \times \nabla_t E_z + k_z \nabla_t H_z) \right] = -\omega\varepsilon E_z \hat{z}$$

$= 0!$ ⇨ **E_z and H_z decouple!**

Dielectric Slab Waveguide (2)

1. TM_z modes

For $0 \le y \le h$:

$$E_{1z} = A_1 \sin k_{yd} y e^{-jk_z z} \qquad k_{yd}^2 + k_z^2 = k_d^2 = \omega^2 \mu_d \varepsilon_d$$

$$E_{1y} = -A_1 \frac{jk_z}{k_{yd}} \cos k_{yd} y e^{-jk_z z}$$

$$H_{1x} = A_1 \frac{j\omega\varepsilon_d}{k_{yd}} \cos k_{yd} y e^{-jk_z z}$$

For $y \ge h$:

$$E_{2z} = A_2 e^{-\alpha y} e^{-jk_z z} \qquad k_z^2 - \alpha^2 = k_0^2 = \omega^2 \mu_0 \varepsilon_0$$

$$E_{2y} = -A_2 \frac{jk_z}{\alpha} e^{-\alpha y} e^{-jk_z z}$$

$$H_{2x} = A_2 \frac{j\omega\varepsilon_0}{\alpha} e^{-\alpha y} e^{-jk_z z}$$

Dielectric Slab Waveguide (3)

At $y = h$:

$$E_{1z} = E_{2z} \quad \Rightarrow \quad A_1 \sin k_{yd} h = A_2 e^{-\alpha h}$$

$$H_{1x} = H_{2x} \quad \Rightarrow \quad A_1 \frac{\varepsilon_d}{k_{yd}} \cos k_{yd} h = A_2 \frac{\varepsilon_0}{\alpha} e^{-\alpha h}$$

Characteristic equation:

$$\frac{k_{yd}}{\varepsilon_d} \tan k_{yd} h = \frac{\alpha}{\varepsilon_0}$$

2. TE_z modes

$$\frac{k_{yd}}{\mu_d} \cot k_{yd} h = -\frac{\alpha}{\mu_0}$$

Solve for α, k_{yd}, and k_z!

Dielectric Slab Waveguide (4)

Definition for cutoff: $\alpha = 0 \Rightarrow k_z = k_0$

At cutoff:

$\tan k_{yd} h = 0$ for TM modes $\Rightarrow k_{yd} h = n\pi \quad n = 0, 1, 2, \ldots$

$\cot k_{yd} h = 0$ for TE modes $\Rightarrow k_{yd} h = (n - \tfrac{1}{2})\pi \quad n = 1, 2, \ldots$

Cutoff frequencies:

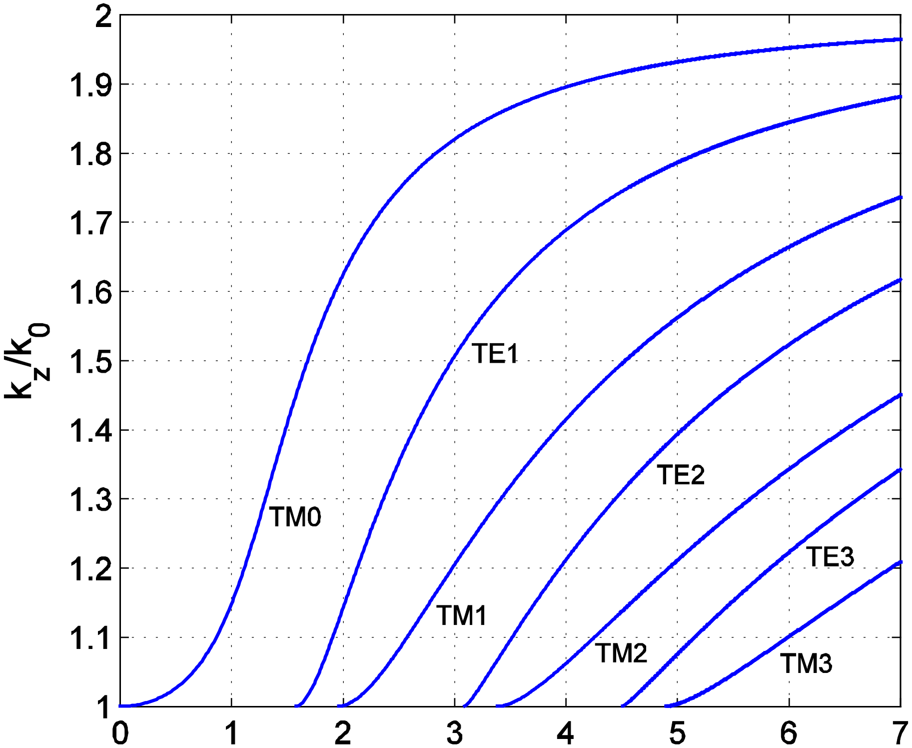
$$\omega_c = \frac{n\pi}{h\sqrt{\mu_d \varepsilon_d - \mu_0 \varepsilon_0}} \qquad n = 0, 1, 2, \ldots \qquad \text{for TM modes}$$

$$\omega_c = \frac{(2n-1)\pi}{2h\sqrt{\mu_d \varepsilon_d - \mu_0 \varepsilon_0}} \qquad n = 1, 2, \ldots \qquad \text{for TE modes}$$

The dominant mode is TM_0, which has no cutoff frequencies!

Dielectric Slab Waveguide (5)

Dispersion curves:



$$k_0 h\sqrt{\varepsilon_{rd} - 1}$$

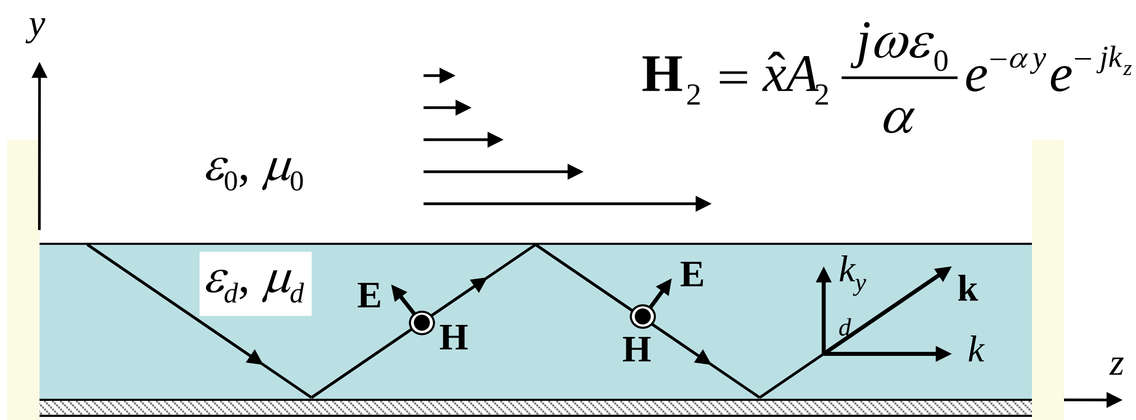
Dielectric Slab Waveguide (6)

Field inside the dielectric slab:

$$\mathbf{H}_1 = \frac{A_1}{2j}\frac{\omega\varepsilon_d}{k_{yd}}\left[-\hat{x}e^{j(k_{yd}y-k_z z)} - \hat{x}e^{-j(k_{yd}y+k_z z)}\right]$$

Superposition of two plane waves!

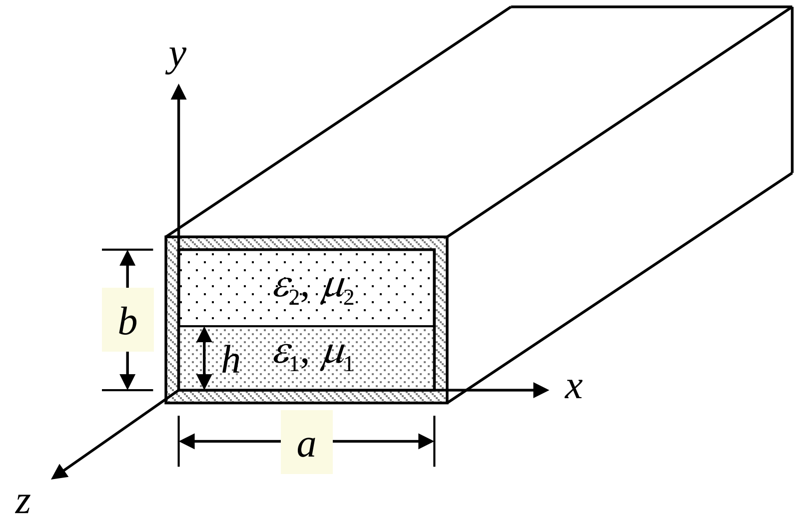
$$\mathbf{H}_2 = \hat{x}A_2\frac{j\omega\varepsilon_0}{\alpha}e^{-\alpha y}e^{-jk_z z}$$



The cutoff corresponds to the internal total reflection!

Partially Filled Waveguide (1)

1. TE_z and TM_z modes exist only when the field has no variation in *x*
2. General modes are hybrid modes
3. At cutoff, hybrid modes reduce to TE_z and TM_z modes



$$\nabla_t \times \left[\frac{1}{k_t^2} (\omega\varepsilon \hat{z} \times \nabla_t E_z + k_z \nabla_t H_z) \right] = -\omega\varepsilon E_z \hat{z}$$

$$\nabla_t \times \left[\frac{1}{k_t^2} (\omega\mu \hat{z} \times \nabla_t H_z - k_z \nabla_t E_z) \right] = -\omega\mu H_z \hat{z}$$

Partially Filled Waveguide (2)

General analysis procedure:

1. Write down the E_z and H_z expressions:

In region 1 ($0 < y < h$):

$$E_{1z} = A_1 \sin k_x x \sin k_{1y} y e^{-jk_z z}$$

$$H_{1z} = B_1 \cos k_x x \cos k_{1y} y e^{-jk_z z}$$

In region 2 ($h < y < b$):

$$E_{2z} = A_2 \sin k_x x \sin k_{2y} (b-y) e^{-jk_z z}$$

$$H_{2z} = B_2 \cos k_x x \cos k_{1y} (b-y) e^{-jk_z z}$$

Dispersion relations

$$k_x = m\pi / a$$

$$k_x^2 + k_{1y}^2 + k_z^2 = k_1^2 = \omega^2 \mu_1 \varepsilon_1$$

$$k_x = m\pi / a$$

$$k_x^2 + k_{2y}^2 + k_z^2 = k_2^2 = \omega^2 \mu_2 \varepsilon_2$$

2. Derive all other field components in the two regions from E_z and H_z

Partially Filled Waveguide (3)

3. Apply the boundary conditions at *y = h*

$$E_{1z}\big|_{y=h} = E_{2z}\big|_{y=h} \qquad H_{1z}\big|_{y=h} = H_{2z}\big|_{y=h}$$

$$E_{1x}\big|_{y=h} = E_{2x}\big|_{y=h} \qquad H_{1x}\big|_{y=h} = H_{2x}\big|_{y=h}$$

4. Find the characteristic equation for non-trivial solutions

$$\left[\frac{\omega\mu_1 k_{1y}}{k_{1t}^2}\tan k_{1y}h + \frac{\omega\mu_2 k_{2y}}{k_{2t}^2}\tan k_{2y}(b-h)\right]$$

$$\times\left[\frac{\omega\varepsilon_1 k_{1y}}{k_{1t}^2}\cot k_{1y}h + \frac{\omega\varepsilon_2 k_{2y}}{k_{2t}^2}\cot k_{2y}(b-h)\right] + \left[k_x k_z\left(\frac{1}{k_{1t}^2} - \frac{1}{k_{2t}^2}\right)\right]^2 = 0$$

$$k_{1t}^2 = k_1^2 - k_z^2 \qquad k_{2t}^2 = k_2^2 - k_z^2$$

Partially Filled Waveguide (4)

5. Solve the characteristic equation

1st set of solution: $$\frac{\mu_1}{k_{1y}}\tan k_{1y}h = -\frac{\mu_2}{k_{2y}}\tan k_{2y}(b-h)$$

2nd set of solution: $$\frac{k_{1y}}{\varepsilon_1}\tan k_{1y}h = -\frac{k_{2y}}{\varepsilon_2}\tan k_{2y}(b-h)$$

6. Analyze the properties of the solutions

For the 1st set of solution: $H_z \to 0$ (TM modes) when $k_z \to 0$ and $m \neq 0$

→ Hybrid EH_{mn} modes ($m \neq 0$)

For the 2nd set of solution: $E_z \to 0$ (TE modes) when $k_z \to 0$ and $m \neq 0$

→ Hybrid HE_{mn} modes ($m \neq 0$)

Partially Filled Waveguide (5)

What happens when $m = 0$?

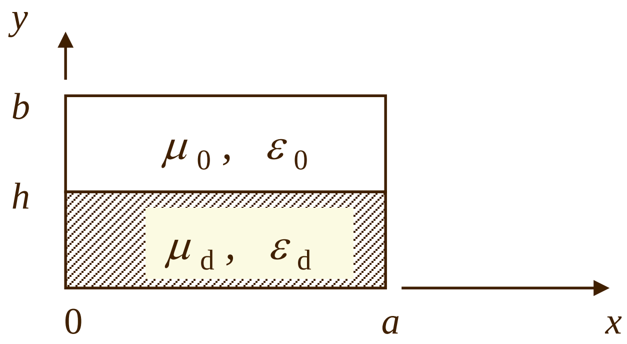
For the 1[st] set of solution: $E_z = 0$ (TE$_{0n}$ modes) for all k_z

TE$_{0n}$ modes (or EH_{0n} modes)

For the 2[nd] set of solution: $E_z = H_z = 0$ (trivial solution) for all k_z

Trivial solution

7. Solve the transcendental equations to compute dispersion curves



$$a/b = 2$$

$$h/b = 0.5$$

$$\varepsilon_r = 4$$

Partially Filled Waveguide (6)

Dispersion curves: