ECE 222b, Homework 1

Problem 1
Starting from the Faraday and Amper-Maxwell laws

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} - \mathbf{M}$$

$$\nabla\times\mathbf{H} = \frac{\partial\mathbf{D}}{\partial t} + \mathbf{J}$$

and from the electric and magnetic continuity equations

$$\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t}$$

$$\nabla\cdot\mathbf{M} = -\frac{\partial\rho_m}{\partial t}$$

derive the electric and magnetic Gauss laws

$$\nabla\cdot\mathbf{D} = \rho$$

$$\nabla\cdot\mathbf{B} = \rho_m$$

Problem 2
Consider two concentric cylinder shells with radiuses *a* and *b* (*b*>*a*). The time-harmonic electric field is given by

$$\mathbf{E} = \begin{cases} 0, & r<a \\ \hat{\varphi}E_0\left(\dfrac{e^{-jk_0(r-b)}}{\sqrt{r}} + \Gamma\dfrac{e^{jk_0(r-b)}}{\sqrt{r}}\right), & a<r<b \\ \hat{\varphi}E_0 T\dfrac{e^{-jk_0(r-b)}}{\sqrt{r}}, & r>b \end{cases}$$

where $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber with ω being a given frequency. The magnetic field is zero for r<a. There exists an electric surface charge density distribution on the shell at *r=b* given via

$$\mathbf{J}_s = \sigma_s\mathbf{E}|_{r=b}, \qquad r=b$$

where $\mathbf{J}_s$ is the surface current at *r=b*; $\mathbf{E}|_{r=b}$ is the field at *r=b*; and σ_s is a constant (surface conductivity). There is no any magnetic surface current at *r=b*.

a) Using the Maxwell equations, calculate the magnetic field. Assume that a,b,r are very large and keep only one dominant (or, in the other words, most slowly decaying) term in the obtained result.

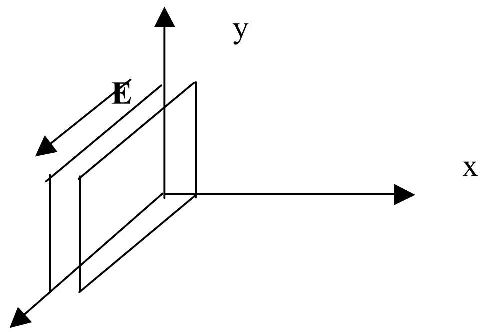
b) Using the boundary conditions at *r=b* for **only** tangential electric and magnetic field components, calculate the values of Γ, T in terms of other given parameters (i.e. $k_0,\omega,\varepsilon_0,\mu_0,\sigma_s$).

c) Using the boundary conditions, give the surface electric and magnetic currents at *r=a*

Problem 3

between the planes is d. The electric field between the planes is given by $\mathbf{E} = \hat{z} A \sin(\pi x / d) \cos(\pi c t / d)$, where A is a constant and c is the wave velocity. Outside the planes both the electric and magnetic fields are zero.

1) Calculate the electric charge density distribution (volume if any and surface)
2) Calculate the magnetic field
3) Calculate the electric current density distribution (volume if any and surface)
4) Consider a closed surface enclosing the PEC plates at $z=0+$ and $z=d-$. Show that, the law of the conservation of energy in the integral form is satisfied. Explain the obtained results.
5) Show that, the law of the conservation of energy in the differential form is satisfied.

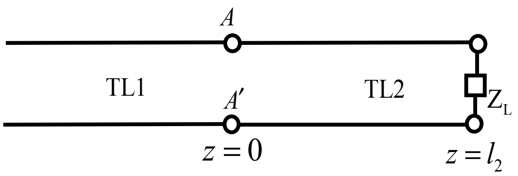


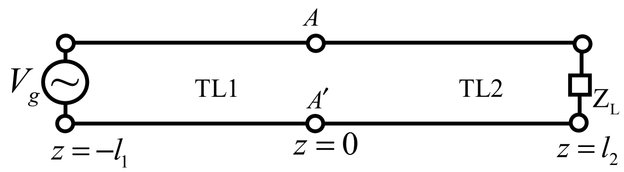
Problem 4

Consider a system, which consists of a transmission line (TL1) connected at port AA' (at $z=0$) to a transmission lines TL2, which is terminated with a load of impedance $Z_L = 8 - j500\,\Omega$ (Fig. 1). The TL1 has an impedance of $Z_1 = 50\,\Omega$. TL2 has an impedance of $Z_2 = 100\,\Omega$ and length $l_2 = 4\,\text{cm}$. TL1 is filled with vacuum, while TL2 is filled with a medium with $\varepsilon_2 = 4\varepsilon_0$, $\mu_2 = \mu_0$.

a) A wave of frequency $f = 3\,\text{GHz}$ and amplitude $V_0 = 10V$ in TL1 is incident on TL2 (Fig. 1(a)). Find the reflection coefficient at AA'. Find the reflected (phasor) voltage and current in TL1.

b) A voltage generator of $\widetilde{V}_g = 10V$ and the same frequency $f = 3\,\text{GHz}$ is connected to TL1 at the point $z = -l_1 = -2.5\,\text{cm}$ as shown in Fig. 1(b). Find the amplitude of the incident voltage wave V_0. Find the time averaged incident power P_{av}^i in TL1 . Find the time averaged reflected power P_{av}^r in TL1 . Find P_{av}^{del}, the power delivered to the load Z_L.





Problem 5

Consider the structure depicted in the Figure below. Two PEC disks of radius *b* reside at *z=0* and *z=d* (*b>>d*). A voltage *V* is applied between the discs. Between the disks a cylinder of radius *a* (*a<b*) is placed. The cylinder is filled with a conducting material with conductivity σ.

a) Find the electric field between the disks for *r<b* (neglect the fringing effect)
b) Find the surface charges and the total charge. Find the capacitance between the PEC disks.
c) Find the magnetic field for *r<b* (use of the cylindrical symmetry of the problem)
d) Find the volume current for *r<b* and the total current. Find the resistance between the PEC disks.
e) Show that the integral conservation of power law is satisfied (take the closed surface enclosing the entire region $\rho < b, 0- < z < d+$. Show that the differential conservation of power law is satisfied.