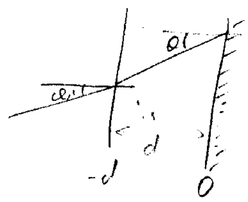
prob 1

for parallel pol $\quad Z_{z0} = \eta_0 \cos\theta_i \quad Z_z = \eta\cos\theta \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$

PEC at $z=0$ $\quad Z\big|_{z=0} = 0$

the impedance at $z=-d$



$$Z\big|_{z=-d} = jZ_z \tan k_z d$$

where $k_z = \beta\cos\theta$

$$\Gamma = \frac{Z\big|_{z=-d} - Z_{z0}}{Z\big|_{z=-d} + Z_{z0}} = \frac{jZ_z \tan k_z d - Z_{z0}}{jZ_z \tan k_z d + Z_{z0}} = \frac{j\eta\cos\theta \tan k_z d - \eta_0\cos\theta_i}{j\eta\cos\theta\tan k_z d + \eta_0\cos\theta_i}$$

and $k_1 \cos\theta_i = k_2\cos\theta$ is the relationship between θ_i & θ.

prob 2

for perpendicular case $\quad Z_{z0} = \eta_0/\cos\theta_i \quad Z_z = \eta/\cos\theta \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$

Similar to prob 1

$$\Gamma = \frac{j\eta/\cos\theta \tan k_z d - \eta_0/\cos\theta_i}{j\eta/\cos\theta \tan k_z d + \eta_0/\cos\theta_i} = \frac{j\eta\cos\theta_i \tan k_z d - \eta_0\cos\theta}{j\eta\cos\theta_i \tan k_z d + \eta_0\cos\theta}.$$

where $\sin\theta = \sqrt{\frac{\epsilon_0\mu_0}{\epsilon_1\mu_1}}\sin\theta_i$

Problem 3

The propagation direction is found from Snell's law:

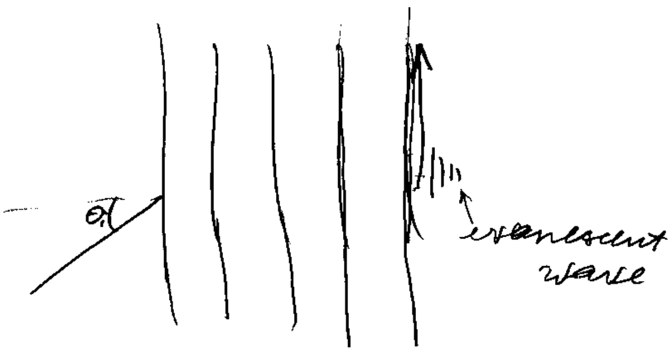
$$\sqrt{\epsilon_n\mu_n}\sin\theta_n = \sqrt{\epsilon_1\mu_1}\sin\theta_i$$

Therefore $\quad \theta_{10} = \sin^{-1}\left\{\sqrt{\frac{\epsilon_1\mu_1}{\epsilon_{10}\mu_{10}}}\sin\theta_i\right\}$

$$\theta_{99} = \sin^{-1}\left\{\sqrt{\frac{\epsilon_1\mu_1}{\epsilon_{99}\mu_{99}}}\sin\theta_i\right\}$$

Note an "interesting" fact that the propagation direction in each region (including the last halfspace) is defined independently of all other regions.

Problem 4



Let us consider TE_z polarization:

The transmitted field is given by:

$$\underline{E}^t = \hat{y}\, T_\perp E_0\, e^{-j\beta_2 (x \sin\theta_t + z\cos\theta_t)}$$

$$\underline{H}^t = (-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)\,\frac{T_\perp E_0}{\eta_2}\, e^{-j\beta_2 (x \sin\theta_t + z\cos\theta_t)}$$

$$\left(\underline{E}^t \times \underline{H}^{t*}\right)\Big|_{z\,\text{component}} = \hat{z}\,\frac{|E_0|^2 |T_\perp|^2}{\eta_2}\,(\cos\theta_t)^*$$

If $\cos\theta_t$ is purely imaginary we do not have any time average power and hence we have the total reflection.
From Problem 3 we know that the direction of propagation in each region totaly defined by the first medium and θ_i. Therefore the condition $\theta_t = \frac{\pi}{2} \Rightarrow \sin\theta_t = 1 \Rightarrow \theta_i = \sin^{-1}\sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}$ leads to total reflection

Problem 5

a)

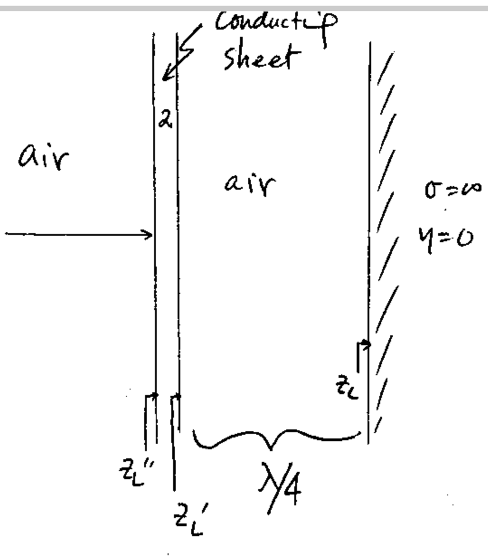
$$Z_L' = \eta_0 \frac{Z_L + j\eta_0 \tan\beta_0 \frac{\lambda}{4}}{\eta_0 + jZ_L \tan\beta_0 \frac{\lambda}{4}}$$

Since $Z_L = 0$

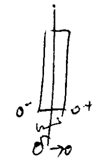
$$Z_L' = j\eta_0 \tan(\beta_0 \tfrac{\lambda}{4})$$

$$= j\eta_0 \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right) \to \infty$$

$$Z_L'' = \eta_2 \frac{Z_L' + j\eta_2 \tan\beta l}{\eta_2 + jZ_L' \tan\beta l}$$



Problem 6



For the energy conservation law on this small area. all the volume integrals vanish because $\delta \to 0$ & all the integrand are limited. So do the surface integrals at the upper & lower surfaces. So what left is the two surface integrals at $z=0^-$ & $z=0^+$

From the textbook (5-14)
for $\perp$ polarization

$$E\big|_{0^-} = 1+\Gamma \qquad E\big|_{0^+} = T$$

$$H\big|_{0^-} = \frac{\cos\theta_i}{\eta_1}(1-\Gamma) \qquad H\big|_{0^+} = \frac{\cos\theta_t}{\eta_2}T$$

where $\Gamma = \dfrac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$ $\qquad T = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$

note now $\cos\theta_t = \sqrt{1-\sin^2\theta_t} = i\sqrt{\sin^2\theta_t - 1} = i\,|\cos\theta_t|$ because $\theta_i > \theta_c$

define $z_2 = \eta_2\cos\theta_i \qquad z_1 = \eta_1|\cos\theta_t| \Rightarrow \eta_1\cos\theta_t = iz_1$

$$\Gamma = \frac{z_2 - iz_1}{z_2 + iz_1} \qquad T = \frac{2z_2}{z_2+iz_1}$$

$$\Gamma = \frac{z_2^2 - z_1^2 - 2iz_1z_2}{z_1^2+z_2^2} \qquad T = \frac{2z_2(z_2 - iz_1)}{z_1^2+z_2^2}$$

$$E\times H\big|_{0^-} = \frac{\cos\theta_i}{\eta_1}(1+\Gamma)(1-\Gamma)^* = \frac{\cos\theta_i}{\eta_1}\left(1-|\Gamma|^2 + 2\,\mathrm{Im}(\Gamma)\right)$$

$$= \frac{\cos\theta_i}{\eta_1}\left[1 - \frac{(z_1^2 - z_2^2)^2 + 4z_1z_2}{(z_1^2+z_2^2)^2} + 2\,\frac{-2iz_1z_2}{z_1^2+z_2^2}\right)$$

if we look at the energy conservation law. there is no power supply $P_s = 0$
no dissipated power $P_d = 0$ So $P_s = 2j\omega(\bar{W}_m - \bar{W}_e)$

You CAN calculate the energy stored in E & M and find the difference. But it is not an easy job. a smart way is to calculate P_s and decide the difference from P_s. if $|P_s| > 0$ $\bar{W}_m > \bar{W}_e$; $|P_s| < 0$ $\bar{W}_m < \bar{W}_e$

field in region ② are plane wave, but they are evanscent.

notice the wave propagate in x. no power loss in x direction

so the integration over the upper & lower surface also cancel each other.

for $\perp$ polarization, in region ②

$$E_y = T_\perp E_0 e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$H_x = -\cos\theta_t \frac{T_\perp}{\eta_2} E_0 e^{-j\beta(x\sin\theta_t + z\cos\theta_t)}$$

$$E_y \times (H_x)^* \Big|_{z=z'} = -\cos\theta_t \frac{|T_\perp|^2}{\eta_2} E_0 e^{-j\beta_2\cos\theta_t z + j\beta z\cos^*\theta_t}$$

know $\cos\theta_t = -j\sqrt{\sin^2\theta - 1}$ because $-j\beta_2\cos\theta_t < 0$ for condition of evanscent wave. ($e^{-\gamma z}$, $\gamma > 0$ will make $z\to\infty$ $E\to 0$)

$$E_y \times (H_x)^* \Big|_{z=z'} = -\cos\theta_t \frac{|T_\perp|^2}{\eta_2} E_0 e^{-2\beta_2|\cos\theta_t| z}$$

$$E_y \times (H_x)^* \Big|_{z=d} - E_y \times (H_x^*) \Big|_{z=0} = -\cos\theta_t \frac{|T_\perp|^2}{\eta_2} E_0^2 (e^{-2\beta_2|\cos\theta_t|d} - 1)$$

$$= -j|\cos\theta_t| \frac{|T_\perp|^2}{\eta_2} E_0^2 (e^{-2\beta_2|\cos\theta_t|d} - 1)$$

$$= j\omega(\bar{W}_m - \bar{W}_e)$$, evanscent $e^{-2\beta_2|\cos\theta_t|d} < 1$

Imaginary part of LHS > 0 so For. $\perp$. $W_m > W_e$

similarly. for $//$ polarization $W_e < W_m$