Problem 1:

The incident electric field is:

$$\mathbf{E}^{inc} = \hat{\mathbf{z}} e^{-jkx} = \hat{\mathbf{z}} \sum_{n=-\infty}^{n=+\infty} j^{-n} J_n(k\rho) e^{jn\varphi} \ .$$

The incident magnetic field is:

$$\mathbf{H}^{inc} = -\hat{\mathbf{y}} \frac{1}{\eta_0} e^{-jkx} = -\hat{\mathbf{y}} \frac{1}{\eta_0} \sum_{n=-\infty}^{n=+\infty} j^{-n} J_n(k\rho) e^{jn\varphi} \ .$$

Since the impedance boundary condition is related to the tangential component, the phi component of the incident magnetic needs to be found:

$$H_\varphi^{inc} = -\frac{j}{\omega\mu} \frac{\partial E_z^{inc}}{\partial \rho} = -\frac{j}{\eta_0} \sum_{n=-\infty}^{n=+\infty} j^{-n} J_n'(k\rho) e^{jn\varphi} \ .$$

The scattered fields are:

$$\begin{cases} \mathbf{E}^{sc} = \hat{\mathbf{z}} \sum_{n=-\infty}^{n=+\infty} a_n H_n^{(2)}(k\rho) e^{jn\varphi} \\ \mathbf{H}^{sc} = -\hat{\varphi} \frac{1}{\eta_0} \sum_{n=-\infty}^{n=+\infty} a_n H_n^{(2)\prime}(k\rho) e^{jn\varphi} \end{cases}$$

So the total field in the tangential directions are:

$$\begin{cases} E_z = E_z^{inc} + E_z^{sc} = \sum_{n=-\infty}^{n=+\infty} \left[j^{-n} J_n(k\rho) + a_n H_n^{(2)}(k\rho) \right] e^{jn\varphi} \\ H_\varphi = H_\varphi^{inc} + H_\varphi^{sc} = -\frac{j}{\eta_0} \sum_{n=-\infty}^{n=+\infty} \left[j^{-n} J_n'(k\rho) + a_n H_n^{(2)\prime}(k\rho) \right] e^{jn\varphi} \end{cases}$$

Enforcing the impedance boundary condition $\frac{E_z}{H_\varphi} = Z_s$, it follows:

$$a_n = -j^{-n} \frac{\eta_0 J_n(ka) + jZ_s J_n'(ka)}{H_n^{(2)}(ka) + jZ_s H_n^{(2)\prime}(ka)} .$$

So the scattered fields are:

$$\begin{cases} \mathbf{E}^{sc} = -\hat{\mathbf{z}} \sum_{n=-\infty}^{n=+\infty} j^{-n} \dfrac{\eta_0 J_n(ka) + jZ_s J_n'(ka)}{H_n^{(2)}(ka) + jZ_s H_n^{(2)\prime}(ka)} H_n^{(2)}(k\rho) e^{jn\varphi} \\ \mathbf{H}^{sc} = \hat{\varphi} \dfrac{1}{\eta_0} \sum_{n=-\infty}^{n=+\infty} j^{-n} \dfrac{\eta_0 J_n(ka) + jZ_s J_n'(ka)}{H_n^{(2)}(ka) + jZ_s H_n^{(2)\prime}(ka)} H_n^{(2)\prime}(k\rho) e^{jn\varphi} - \dfrac{\hat{\varphi}}{\eta_0 \rho} \sum_{n=-\infty}^{n=+\infty} jnj^{-n} \dfrac{\eta_0 J_n(ka) + jZ_s J_n'(ka)}{H_n^{(2)}(ka) + jZ_s H_n^{(2)\prime}(ka)} H_n^{(2)}(k\rho) e^{jn\varphi} \end{cases}$$

Problem 2

TMz case:

$\mathbf{E}^{inc} = \mathbf{z} e^{-jkx}$,

$$\begin{cases} E_z^{tot} = \sum_{n=-\infty}^{n=+\infty} \left[j^{-n} J_n(k\rho) + a_n H_n^{(2)}(k\rho) \right] e^{jn\varphi} \\ H_\varphi^{tot} = \dfrac{-j}{\eta_0} \sum_{n=-\infty}^{n=+\infty} \left[j^{-n} J_n'(k\rho) + a_n H_n^{(2)\prime}(k\rho) \right] e^{jn\varphi} \end{cases}$$

where $a_n = -\dfrac{j^{-n} J_n(ka)}{H_n^{(2)}(ka)}$. Then the current on the surface of the PEC cylinder is:

$J_{Sz} = \dfrac{-j}{\eta_0} \sum_{n=-\infty}^{n=+\infty} \left[j^{-n} J_n'(ka) - \dfrac{j^{-n} J_n(ka)}{H_n^{(2)}(ka)} H_n^{(2)\prime}(ka) \right] e^{jn\varphi}$. The current flows in the z direction.

TEz case:

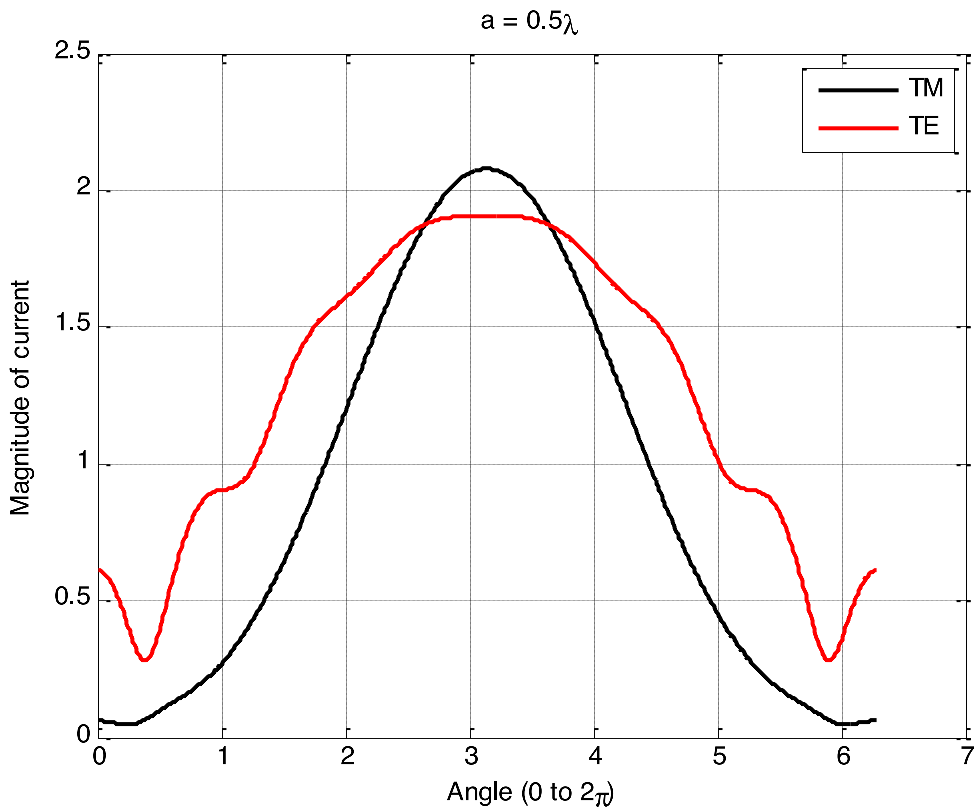
The incident magnetic field is $\mathbf{H}^{inc} = \mathbf{z} e^{-jkx}$. The total fields in the tangential directions are:

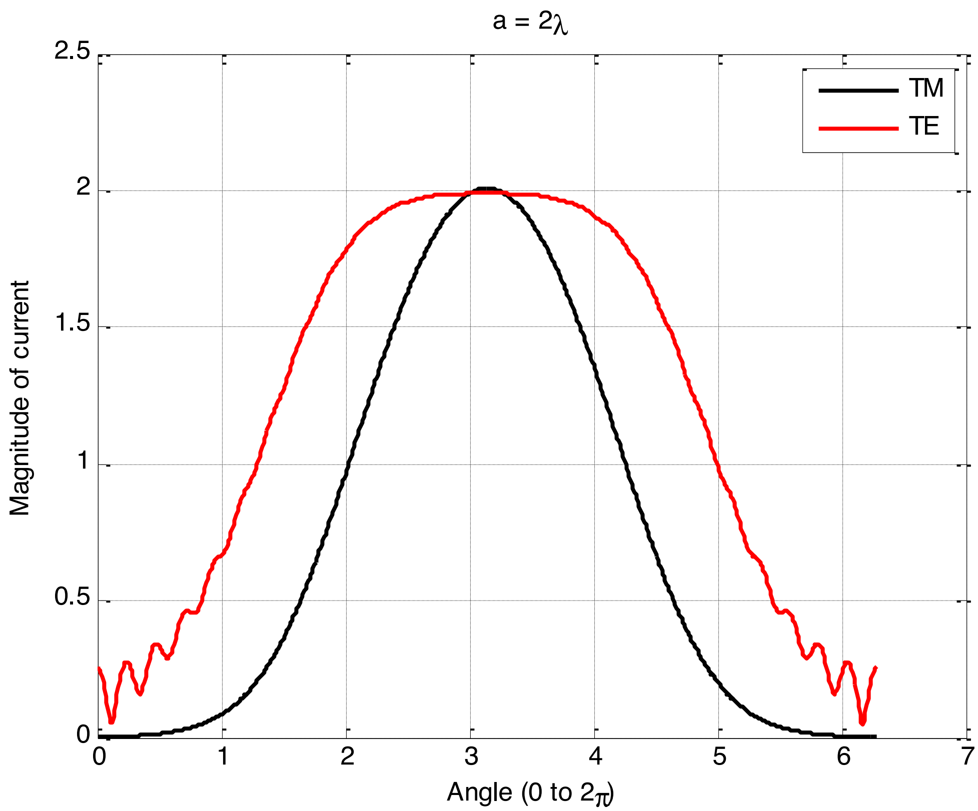
$$\begin{cases} H_z^{tot} = \sum_{n=-\infty}^{n=+\infty} \left[j^{-n} J_n(k\rho) + b_n H_n^{(2)}(k\rho) \right] e^{jn\varphi} \\ E_\varphi^{tot} = j\eta_0 \sum_{n=-\infty}^{n=+\infty} \left[j^{-n} J_n'(k\rho) + b_n H_n^{(2)\prime}(k\rho) \right] e^{jn\varphi} \end{cases}.$$

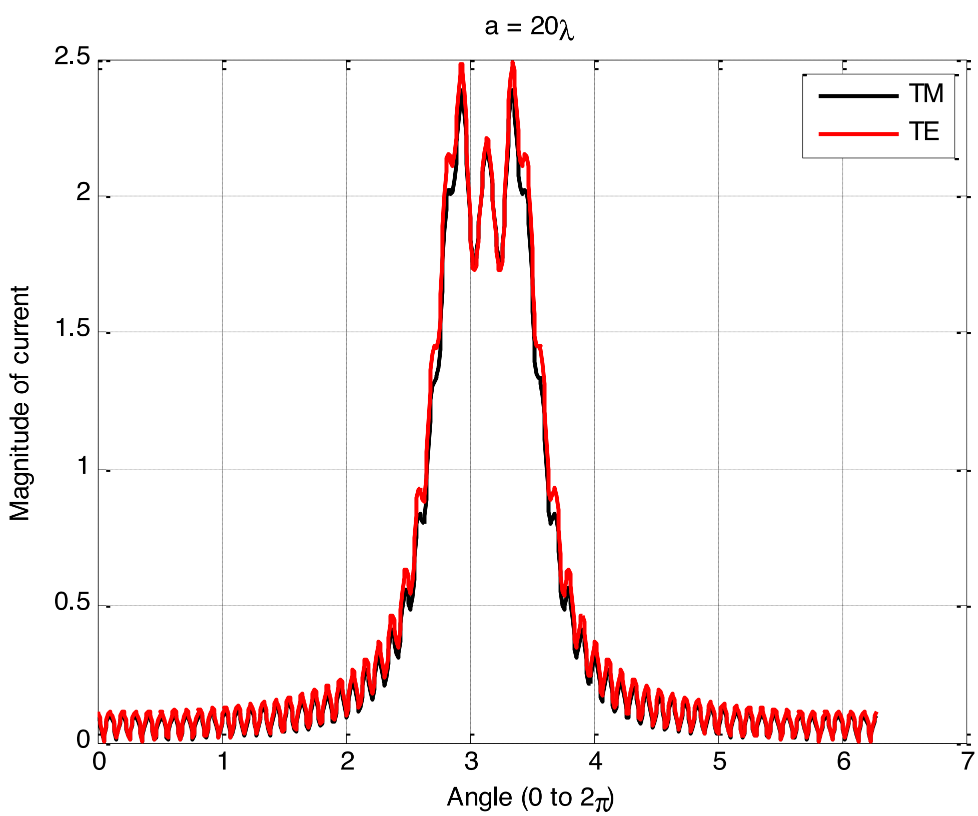
where $b_n = -\frac{j^{-n} J_n'(ka)}{H_n^{(2)\prime}(ka)}$.

The total electric current is flowing in the phi direction, and its magnitude is:

$$J_{S\varphi} = -\sum_{n=-\infty}^{n=+\infty} \left[j^{-n} J_n(k\rho) - \frac{j^{-n} J_n'(ka)}{H_n^{(2)\prime}(ka)} H_n^{(2)}(k\rho) \right] e^{jn\varphi}.$$




a = 2λ
TM
TE
Magnitude of current
Angle (0 to 2π)


a = 20λ
TM
TE
Magnitude of current
Angle (0 to 2π)