ECE 107: Homework 8

1)  

5.26  A uniform current density given by

\[ \mathbf{J} = \hat{z} J_0 \quad (\text{A/m}^2) \]

gives rise to a vector magnetic potential

\[ \mathbf{A} = -\hat{z} \frac{\mu_0 J_0}{4} (x^2 + y^2) \quad (\text{Wb/m}) \]

(b) Use the expression for \( \mathbf{A} \) to find \( \mathbf{H} \).

(c) Use the expression for \( \mathbf{J} \) in conjunction with Ampère’s law to find \( \mathbf{H} \). Compare your result with that obtained in part (b).

2)  

6.19*  If the current density in a conducting medium is given by

\[ \mathbf{J}(x, y, z; t) = (\hat{x} z - \hat{y} 3y^2 + \hat{z} 2x) \cos \omega t \]

determine the corresponding charge distribution \( \rho_c(x, y, z; t) \).

3)
6.16 The parallel-plate capacitor shown in Fig. 6-25 is filled with a lossy dielectric material of relative permittivity \(\varepsilon_r\) and conductivity \(\sigma\). The separation between the plates is \(d\) and each plate is of area \(A\). The capacitor is connected to a time-varying voltage source \(V(t)\).

(a) Obtain an expression for \(I_c\), the conduction current flowing between the plates inside the capacitor, in terms of the given quantities.

(b) Obtain an expression for \(I_d\), the displacement current flowing inside the capacitor.

(c) Based on your expressions for parts (a) and (b), give an equivalent-circuit representation for the capacitor.

(d) Evaluate the values of the circuit elements for \(A = 4\) cm\(^2\), \(d = 0.5\) cm, \(\varepsilon_r = 4\), \(\sigma = 2.5\) (S/m), and \(V(t) = 10\cos(3\pi \times 10^4 t)\) (V).

4)

6.6 The square loop shown in Fig. 6-19 is coplanar with a long, straight wire carrying a current

\[ I(t) = 5 \cos 2\pi \times 10^4 t \quad (A) \]

(a) Determine the emf induced across a small gap created in the loop.
(b) Determine the direction and magnitude of the current that would flow through a 4-Ω resistor connected across the gap. The loop has an internal resistance of 1 Ω.

Figure 6-19: Loop coplanar with long wire (Problem 6.6).