1. Consider a three-section transmission line with the characteristic impedances $Z_{01}, Z_{02}, Z_{03}$ as defined in Fig. 1. The rightmost section is infinite and there is no wave propagating from right to left in it. The length of Section 2 is $l_2$. The wavenumber is $\beta$ in all transmission lines.

   a. Give the input impedance $Z_{in}(z_2)$ at point $z_2$.
   
   b. Give the length $l_2$ and the characteristic impedance $Z_{02}$ that lead to no reflection in the leftmost transmission line.

2. Consider the system of a sphere and an infinitely long cylinder filled with a constant volume charge density $\rho_0$, as shown in Fig. 2. The sphere is centered at $(x_1 = 0, y_1 = 0, z_1 = 0)$ and its radius is $R_1 = 2\text{ cm}$. The permittivity of the entire space is that of vacuum. The cylinder’s radius is $R_2 = 1\text{ cm}$ and its axis is centered at $(x_2 = 4, y_2 = 0)$. Find the electrostatic field. It is enough to give the field only outside the charged (sphere and cylinder) domains. Giving the field also inside the charged domains is a bonus.
Problem 1.

a) The input impedance at point of $z_2$ is $Z_{in}(z_2) = Z_0$ because there is no wave going from right to left and there is no standing wave. The input impedance coincides with the characteristic impedance in this case because the total voltage and current coincides with the rightward going voltage and current.

b) The rightmost transmission line is lossless and the impedance

c) The impedance $Z_{in}(z_2)$ seen as the input impedance of line 3 can also be seen as the load impedance at the right end of line 2. Since this impedance is real, one can use a quarter wave transformer to match line 1 with the rest. Therefore, the length of line 2 should be of quarter wavelength $l_2 = \lambda/4$ with $\lambda = 2\pi/\beta$ and the characteristic impedance of $Z_{02} = \sqrt{Z_{01}Z_{03}}$.

Problem 2.

The sphere and cylinder both have symmetries and therefore one can use Gauss’s law for finding their fields. The field due to the entire system is obtained as a superposition of the fields due to the sphere and cylinder.

The field $E_1$ due to the sphere charge can be calculated by understanding that, due to the symmetry, $E_1 = \hat{R}E(R)$ and choosing the sphere of radius $R$ inside and outside the sphere, leading to

$$4\pi R^2 \varepsilon_0 E_{1|R} = \frac{4}{3} \pi R^3 \rho_0 \Rightarrow E_{1|R} = \frac{\rho_0 R}{3\varepsilon_0} \hat{R}, \quad R \leq R_1$$

$$4\pi R^2 \varepsilon_0 E_{1|R} = \frac{4}{3} \pi R^3 \rho_0 \Rightarrow E_{1|R} = \frac{R^3 \rho_0}{3\varepsilon_0 R^2} \hat{R}, \quad R > R_1$$

where $R = \sqrt{x^2 + y^2 + z^2}$ and $\hat{R} = \frac{\hat{x} + \hat{y} + \hat{z}}{R}$. 
Similarly, for the field due to the cylinder, $\mathbf{E}_2 = \hat{r}_2 E_{r_2}(r_2)$, where $r_2 = \sqrt{(x-x_2)^2 + y^2}$ and $
abla_2 = \left((x-x_2)\hat{x} + y\hat{y}\right)/r_2$ is the radial unit vector with the respect to the axis at $(x_2 = 4, y_2 = 0)$. Choosing a cylindrical surface of an arbitrary height $l$ and radius $r_2$ in the Gauss’ law results in

$$2\pi r_2 l \varepsilon_0 E_{r_2} = \pi r_2^2 \rho_0 \Rightarrow E_{r_2} = \frac{\rho_0 r_2}{2\varepsilon_0}, \quad r_2 \leq R_2$$

$$2\pi r_2 l \varepsilon_0 E_{r_2} = \pi R_2^2 \rho_0 \Rightarrow E_{r_2} = \frac{R_2^2 \rho_0}{2\varepsilon_0 r_2}, \quad r_2 > R_2$$

The total electrostatic field is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$. 